United Kingdom
Mathematics Trust

British Mathematical Olympiad

Round 1

Friday 30 November 2018

Instructions

1. Time allowed: $3\frac{1}{2}$ hours.
2. Full written solutions – not just answers – are required, with complete proofs of any assertions you may make. Marks awarded will depend on the clarity of your mathematical presentation. Work in rough first, and then write up your best attempt. Do not hand in rough work.
3. One complete solution will gain more credit than several unfinished attempts. It is more important to complete a small number of questions than to try all the problems.
4. Each question carries 10 marks. However, earlier questions tend to be easier. In general you are advised to concentrate on these problems first.
5. The use of rulers, set squares and compasses is allowed, but calculators and protractors are forbidden.
6. Start each question on a fresh sheet of paper. Write on one side of the paper only. On each sheet of working write the number of the question in the top left hand corner and your name, initials and school in the top right hand corner.
7. Complete the cover sheet provided and attach it to the front of your script, followed by your solutions in question number order.
8. Staple all the pages neatly together in the top left hand corner.
9. To accommodate candidates sitting in other time zones, please do not discuss the paper on the internet until 8am GMT on Saturday 1 December when the solutions video will be released at https://bmos.ukmt.org.uk
10. **Do not turn over until told to do so.**

Enquiries about the British Mathematical Olympiad should be sent to:

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1. A list of five two-digit positive integers is written in increasing order on a blackboard. Each of the five integers is a multiple of 3, and each digit 0,1,2,3,4,5,6,7,8,9 appears exactly once on the blackboard. In how many ways can this be done? *Note that a two-digit number cannot begin with the digit 0.*

2. For each positive integer $n \geq 3$, we define an n-ring to be a circular arrangement of n (not necessarily different) positive integers such that the product of every three neighbouring integers is n. Determine the number of integers n in the range $3 \leq n \leq 2018$ for which it is possible to form an n-ring.

3. Ares multiplies two integers which differ by 9. Grace multiplies two integers which differ by 6. They obtain the same product T. Determine all possible values of T.

4. Let Γ be a semicircle with diameter AB. The point C lies on the diameter AB and points E and D lie on the arc BA, with E between B and D. Let the tangents to Γ at D and E meet at F. Suppose that $\angle ACD = \angle ECB$.

Prove that $\angle EFD = \angle ACD + \angle ECB$.

5. Two solid cylinders are mathematically similar. The sum of their heights is 1. The sum of their surface areas is 8π. The sum of their volumes is 2π. Find all possibilities for the dimensions of each cylinder.

6. Ada the ant starts at a point O on a plane. At the start of each minute she chooses North, South, East or West, and marches 1 metre in that direction. At the end of 2018 minutes she finds herself back at O. Let n be the number of possible journeys which she could have made. What is the highest power of 10 which divides n?